

### **OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

# MATHEMATICS

4726

**Further Pure Mathematics 2** 

### **Specimen Paper**

Additional materials: Answer booklet Graph paper List of Formulae (MF 1)

**TIME** 1 hour 30 minutes

# **INSTRUCTIONS TO CANDIDATES**

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures, unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphic calculator in this paper.

## INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- You are reminded of the need for clear presentation in your answers.

- 1 (i) Starting from the definition of  $\cosh x$  in terms of  $e^x$ , show that  $\cosh 2x = 2\cosh^2 x 1$ . [2]
  - (ii) Given that  $\cosh 2x = k$ , where k > 1, express each of  $\cosh x$  and  $\sinh x$  in terms of k. [4]



The diagram shows the graph of

2

$$y = \frac{2x^2 + 3x + 3}{x + 1}.$$

- (i) Find the equations of the asymptotes of the curve.
- (ii) Prove that the values of y between which there are no points on the curve are -5 and 3. [4]
- 3 (i) Find the first three terms of the Maclaurin series for  $\ln(2+x)$ . [4]
  - (ii) Write down the first three terms of the series for  $\ln(2-x)$ , and hence show that, if x is small, then

$$\ln\left(\frac{2+x}{2-x}\right) \approx x \,. \tag{3}$$

[3]

4 The equation of a curve, in polar coordinates, is

$$r = 2\cos 2\theta \qquad (-\pi < \theta \leqslant \pi)$$

(i) Find the values of  $\theta$  which give the directions of the tangents at the pole.

One loop of the curve is shown in the diagram.



(ii) Find the exact value of the area of the region enclosed by the loop.



The diagram shows the curve  $y = \frac{1}{x+1}$  together with four rectangles of unit width.

(i) Explain how the diagram shows that

5

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} < \int_0^4 \frac{1}{x+1} \, \mathrm{d}x \,.$$
<sup>[2]</sup>

The curve  $y = \frac{1}{x+2}$  passes through the top left-hand corner of each of the four rectangles shown.

- (ii) By considering the rectangles in relation to this curve, write down a second inequality involving  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$  and a definite integral. [2]
- (iii) By considering a suitable range of integration and corresponding rectangles, show that

$$\ln(500.5) < \sum_{r=2}^{1000} \frac{1}{r} < \ln(1000) .$$
[4]

#### [Turn over

[5]

[3]

6 (i) Given that  $I_n = \int_0^1 x^n \sqrt{1-x} \, dx$ , prove that, for  $n \ge 1$ ,

$$(2n+3)I_n = 2nI_{n-1}.$$
 [6]

[4]

- (ii) Hence find the exact value of  $I_2$ .
- 7 The curve with equation

$$y = \frac{x}{\cosh x}$$

has one stationary point for x > 0.

(i) Show that the *x*-coordinate of this stationary point satisfies the equation  $x \tanh x - 1 = 0$ . [2]

The positive root of the equation  $x \tanh x - 1 = 0$  is denoted by  $\alpha$ .

- (ii) Draw a sketch showing (for positive values of x) the graph of  $y = \tanh x$  and its asymptote, and the graph of  $y = \frac{1}{x}$ . Explain how you can deduce from your sketch that  $\alpha > 1$ . [3]
- (iii) Use the Newton-Raphson method, taking first approximation  $x_1 = 1$ , to find further approximations  $x_2$  and  $x_3$  for  $\alpha$ . [5]
- (iv) By considering the approximate errors in  $x_1$  and  $x_2$ , estimate the error in  $x_3$ . [3]
- 8 (i) Use the substitution  $t = \tan \frac{1}{2}x$  to show that

$$\int_{0}^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}} \, \mathrm{d}x = 2\sqrt{2} \int_{0}^{1} \frac{t}{(1+t)(1+t^2)} \, \mathrm{d}t \,.$$
 [4]

(ii) Express 
$$\frac{t}{(1+t)(1+t^2)}$$
 in partial fractions. [5]

(iii) Hence find 
$$\int_{0}^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos x}{1+\sin x}} \, dx$$
, expressing your answer in an exact form. [4]

				1		
1	(i)	RHS = 2(	$\left(\frac{1}{2}(e^{x}+e^{-x})\right)^{2}-1=\frac{1}{2}(e^{2x}+e^{-2x})=LHS$	M1		For correct squaring of $(e^x + e^{-x})$
		`	,	A1	2	For completely correct proof
	(ii)	$2\cosh^2 x$	$-1 = k \Longrightarrow \cosh x = \sqrt{\left(\frac{1}{2}(1+k)\right)}$	M1		For use of (i) and solving for $\cosh x$
				A1		For correct positive square root only
		$2\sinh^2 x +$	$1 = k \Longrightarrow \sinh x = \pm \sqrt{\left(\frac{1}{2}(k-1)\right)}$	M1		For use of $\cosh^2 x - \sinh^2 x = 1$ , or equivalent
				A1	4	For both correct square roots
					6	
2	(i)	x = -1 is a	an asymptote	B1		For correct equation of vertical asymptote
		y = 2x + 1	$+\frac{2}{r+1}$	M1		For algebraic division, or equivalent
		Hence $y =$	2x+1 is an asymptote	A1	3	For correct equation of oblique asymptote
	(ii)	EITHER:	Quadratic $2x^2 + (3-y)x + (3-y) = 0$	M1		For using discriminant of relevant quadratic
			has no real roots if $(3-y)^2 < 8(3-y)$	A1		For correct inequality or equation in y
			Hence $(3-y)(-5-y) < 0$	M1		For factorising, or equivalent
			So required values are 3 and $-5$	A1		For given answer correctly shown
		OR:	$\frac{dy}{dx} = 2 - \frac{2}{(x+1)^2} = 0$	M1		For differentiating and equating to zero
			Hence $(x+1)^2 = 1$	A1		For correct simplified quadratic in $x$
			So $x = -2$ and $0 \Rightarrow y = -5$ and 3	M1		For solving for x and substituting to find y
				A1	4	For given answer correctly shown
					7	
3	(i)	EITHER:	If $f(x) = \ln(x+2)$ , then $f'(x) = \frac{1}{2+x}$	M1		For at least one differentiation attempt
			and $f''(x) = -\frac{1}{(2+x)^2}$	A1		For correct first and second derivatives
			$f(0) = \ln 2, f'(0) = \frac{1}{2}, f''(0) = -\frac{1}{4}$	A1√		For all three evaluations correct
			Hence $\ln(x+2) = \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$	A1		For three correct terms
		OR:	$\ln(2+x) = \ln[2(1+\frac{1}{2}x)]$	M1		For factorising in this way
			$= \ln 2 + \ln(1 + \frac{1}{2}x)$	A1		For using relevant log law correctly
			$(\frac{1}{2}x)^2$	MI		For use of standard series surrousing
			$= \ln 2 + \frac{1}{2}x - \frac{1}{2} + \dots$	INI I		For use of standard series expansion
			$= \ln 2 + \frac{1}{2}x - \frac{1}{8}x^2 + \dots$	A1	4	For three correct terms
	(ii)	$\ln(2-x) \approx$	$x \ln 2 - \frac{1}{2}x - \frac{1}{8}x^2$	B1√		For replacing x by $-x$
		$\ln\left(\frac{2+x}{2-x}\right)$	$\approx (\ln 2 + \frac{1}{2}x - \frac{1}{8}x^2) - (\ln 2 - \frac{1}{2}x - \frac{1}{8}x^2)$	M1		For subtracting the two series
		()	$\approx x$ , as required	A1	3	For showing given answer correctly
					7	

$\begin{array}{c c c c c c c c c c c c c c c c c c c $				1		
A1A1For any two correct values A1(ii) Area is $\frac{1}{2} \int_{-\frac{1}{2}\pi}^{1/2} 4\cos^2 2\theta  d\theta$ M1For us of correct formula $\frac{1}{2} \int r^2  d\theta$ i.e. $\int_{-\frac{1}{2}\pi}^{1/2} 1 + \cos 4\theta  d\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{2}\pi}^{1/2} = \frac{1}{2}\pi$ M1For using double-angle formulaA1So for errect initis from (1)For errect initis from (1)5(i) LHS is the total area of the four rectangles RHS is the corresponding area under the curve, which is clearly greaterB16(ii) $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} > \int_{0}^{d} \frac{1}{x+2}  dx$ M1For identifying rectangle areas (not heights)7(iii) $\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{d} \frac{1}{x+2}  dx$ M1For attempt at relevant new inequality(iii) $\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{d} \frac{1}{x+2}  dx$ M1For statement(iii) Sum is the area of 999 rectangles Bounds are $\int_{0}^{999} \frac{1}{x+2}  dx$ and $\int_{0}^{999} \frac{1}{x+1}  dx$ M1For stating either integral as a bound8So lower bound is $[\ln(x+1)]_{0}^{999} = \ln(1000)$ A1For using integration by parts6(i) $I_n = \left[-\frac{2}{3}x^n(1-x)^2 \int_{0}^{1} + \frac{2}{3}n \int_{0}^{1} x^{n-1}(1-x)^{\frac{3}{2}}  dx$ M1For using integration by parts $=\frac{2}{3}n \int_{0}^{1} x^{n-1}(1-x) \sqrt{(1-x)}  dx$ M1For soluting integration by parts(ii) $I_n = \left[-\frac{2}{3}x^n(1-x)^2 \int_{0}^{1} + \frac{2}{3}n \int_{0}^{1} \frac{x^{n-1}}{x^n} + \frac{1}{x^n} + \frac$	4	(i)	$r = 0 \Longrightarrow \cos 2\theta = 0 \Longrightarrow \theta = \pm \frac{1}{4}\pi, \pm \frac{3}{4}\pi$	M1		For equating <i>r</i> to zero and solving for $\theta$
$\begin{array}{ c c c c c } \hline  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c c  &  c $				A1		For any two correct values
(ii) Area is $\frac{1}{2} \int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 4\cos^2 2\theta  d\theta$ i.e. $\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 1 + \cos 4\theta  d\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} = \frac{1}{2}\pi$ All For using double-angle formula All For $\theta + \frac{1}{4}\sin 4\theta$ correct Al For $\theta + \frac{1}{4}\sin 4\theta$ correct (ii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{4} \frac{1}{\pi + 2}  dx$ (ii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{4} \frac{1}{\pi + 2}  dx$ (iii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{4} \frac{1}{\pi + 2}  dx$ (iii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2} > \int_{0}^{4} \frac{1}{\pi + 2}  dx$ (iii) Sum is the area of 999 rectangles Bounds are $\int_{0}^{999} \frac{1}{\pi + 2}  dx$ and $\int_{0}^{999} \frac{1}{\pi + 1}  dx$ So lower bound is $[\ln(x + 2)]_{0}^{999} = \ln(500.5)$ and upper bound is $[\ln(x + 1)]_{0}^{999} = \ln(1000)$ Al For showing the given value correctly Al $\frac{1}{2}$ For correct flats tage result For correct flats tage result For using integration by parts Al For soluting the remaining integral up For correct flats tage result For use of limits in integrated term For use of limits in integrated term For soluting the remaining integral up For correct flation between $I_{e}$ and $I_{e-1}$ Hence $(2\pi + 3)I_{e} = 2\pi I_{e-1}^{-1}$ , as required (iv) $I_{e} = \frac{1}{2\pi} \left[-\frac{2}{3}(1 - x)^{2}\right]_{0}^{1} = \frac{1}{105}$ Al For volues of the recurrence relation For correct answer (iv) $I_{e} = \frac{1}{2\pi} \left[-\frac{2}{3}(1 - x)^{2}\right]_{0}^{1} = \frac{1}{105}$ Al For correct answer				AI	3	For all four correct values and no others
$i.e. \int_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} 1 + \cos 4\theta  d\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{\frac{1}{2}\pi}^{\frac{1}{2}\pi} = \frac{1}{2}\pi$ $II = \begin{cases} For correct limits from (i) For using double-angle formula For \theta + \frac{1}{2}\sin 4\theta correct For correct (exact) answer For \frac{1}{2}\sin 4\theta correct (exact) answer For \frac{1}{2}\sin 4\theta correct (exact) answer For \frac{1}{2}\sin 4\theta correct (exact) answer For correct (exact) and For showing the given value correctly For showing the given value correctly For showing the given value correctly For use for the exact for formula for the formula for the formula for the four formula for the four formula for for splitting the remaining integral up For correct first stage result For use of the returned formula formula for for correct expression in terms of I_0 for correct answer Formula formula formula formula for expression in terms of I_0 for correct answer Formula formu$		(ii)	Area is $\frac{1}{2} \int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 4\cos^2 2\theta \mathrm{d}\theta$	M1		For us of correct formula $\frac{1}{2}\int r^2 d\theta$
i.e. $\int_{-\frac{1}{2}}^{\frac{1}{2}} 1 + \cos 4\theta  d\theta = \left[\theta + \frac{1}{4} \sin 4\theta\right]_{\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{2}\pi$ Al				B1√		For correct limits from (i)
A1 A1For $\theta + \frac{1}{4} \sin 4\theta$ correct For correct (exact) answer5(i) LHS is the total area of the four rectangles RHS is the corresponding area under the curve, which is clearly greaterB1For identifying rectangle areas (not heights) B16(ii) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{3} > \int_{0}^{4} \frac{1}{x + 2} dx$ M1For attempt at relevant new inequality A17(iii) Sum is the area of 999 rectangles B0 ounds are $\int_{0}^{99} \frac{1}{x + 2} dx$ and $\int_{0}^{999} \frac{1}{x + 1} dx$ M1For correct statement880 ounds are $\int_{0}^{990} \frac{1}{x + 2} dx$ and $\int_{0}^{999} \frac{1}{x + 1} dx$ M1For stating either integral as a bound880 ounds are $\int_{0}^{990} \frac{1}{x + 2} dx$ and $\int_{0}^{999} \frac{1}{x + 1} dx$ M1For stating either integral as a bound98910000)A1499101000)A1499101000)A1499101000A199101000A1991010009149101019919119119119119119119119119119119119119119119<			i.e. $\int_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} 1 + \cos 4\theta  \mathrm{d}\theta = \left[\theta + \frac{1}{4}\sin 4\theta\right]_{-\frac{1}{4}\pi}^{\frac{1}{4}\pi} = \frac{1}{2}\pi$	M1		For using double-angle formula
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$ \boxed{8} $ $ \boxed{8} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{9} $ $ \boxed{1} $ $ \boxed{9} $				A1	5	For correct (exact) answer
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A1 4 For correct answer			Hence $I_2 = \frac{8}{35} \left[ -\frac{2}{3} (1-x)^{\frac{3}{2}} \right]_0^1 = \frac{16}{105}$	M1		For evaluation of $I_0$
10				A1	4	For correct answer
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	1					

7	(i)	$\frac{dy}{dx} = \frac{\cosh x - x \sinh x}{\cosh^2 x}$	M1		For differentiating and equating to zero
		Max occurs when $\cosh x = x \sinh x$ , i.e. $x \tanh x = 1$	A1	2	For showing given result correctly
	(ii)			3	For correct sketch of $y = \tanh x$ For identification of asymptote $y = 1$ For correct explanation of $\alpha > 1$ based on intersection (1, 1) of $y = 1/x$ with $y = 1$
	(iii)	$x_{n+1} = x_n - \frac{x_n \tanh x_n - 1}{\tanh x_n + x_n \operatorname{sech}^2 x_n}$	M1		For correct Newton-Raphson structure
			A1		For all details in $x - \frac{f(x)}{f'(x)}$ correct
		$x_1 = 1 \Longrightarrow x_2 = 1.20177$	M1 A1		For using Newton-Raphson at least once For $x_2$ correct to at least 3sf
		<i>x</i> <sub>3</sub> = 1.1996785	A1	5	For $x_3$ correct to at least 4sf
	(iv)	$e_1 \approx 0.2, \ e_2 \approx -0.002$	B1√		For both magnitudes correct
		$\frac{e_3}{e_2^2} \approx \frac{e_2}{e_1^2} \Longrightarrow e_3 \approx -2 \times 10^{-7}$	M1		For use of quadratic convergence property
			A1	3	For answer of correct magnitude
				13	
				10	
_		dt , 2			
8	(i)	$\frac{\mathrm{d}t}{\mathrm{d}x} = \frac{1}{2}(1+t^2)$	B1		For this relation, stated or used
8	(i)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$	B1 M1		For this relation, stated or used For complete substitution for <i>x</i> in integrand
8	(i)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$	B1 M1 B1		For this relation, stated or used For complete substitution for $x$ in integrand For justification of limits 0 and 1 for $t$
8	(i)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$	B1 M1 B1 A1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer
8	(i) (ii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$	B1 M1 B1 A1 B1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs
8	(i) (ii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$	B1 M1 B1 A1 B1 M1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i>
8	(i) (ii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$	B1 M1 B1 A1 B1 M1 B1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i>
8	(i) (ii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$	<ul> <li>B1</li> <li>M1</li> <li>B1</li> <li>A1</li> <li>B1</li> <li>M1</li> <li>B1</li> <li>A1</li> <li>A1</li> <li>A1</li> </ul>	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i>
8	(i) (ii) (iii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$ Int is $2\sqrt{2} \left[ -\frac{1}{2} \ln(1+t) + \frac{1}{4} \ln(1+t^2) + \frac{1}{2} \tan^{-1}t \right]_0^1$	B1 M1 B1 A1 B1 M1 B1 A1 A1 B1√	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i> For both logarithm terms correct
8	(i) (ii) (iii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$ Int is $2\sqrt{2} \left[ -\frac{1}{2} \ln(1+t) + \frac{1}{4} \ln(1+t^2) + \frac{1}{2} \tan^{-1}t \right]_0^1$	B1 M1 B1 A1 B1 M1 B1 A1 A1 B1√ B1√	5	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i> For both logarithm terms correct For the inverse tan term correct
8	(i) (ii) (iii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$ Int is $2\sqrt{2} \left[ -\frac{1}{2} \ln(1+t) + \frac{1}{4} \ln(1+t^2) + \frac{1}{2} \tan^{-1}t \right]_0^1$ $= \frac{1}{4} (\pi - 2\ln 2) \sqrt{2}$	B1 M1 B1 A1 B1 M1 B1 A1 A1 A1 B1√ B1√ M1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i> For both logarithm terms correct For the inverse tan term correct For use of appropriate limits
8	(i) (ii) (iii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$ Int is $2\sqrt{2} \left[ -\frac{1}{2} \ln(1+t) + \frac{1}{4} \ln(1+t^2) + \frac{1}{2} \tan^{-1}t \right]_0^1$ $= \frac{1}{4}(\pi - 2\ln 2)\sqrt{2}$	B1 M1 B1 A1 B1 M1 B1 A1 A1 B1√ B1√ M1 A1	4 5 _4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i> For both logarithm terms correct For the inverse tan term correct For use of appropriate limits For correct (exact) answer in any form
8	(i) (ii) (iii)	$\frac{dt}{dx} = \frac{1}{2}(1+t^2)$ $\int_0^{\frac{1}{2}\pi} \sqrt{\frac{1-\cos}{1+\sin x}}  dx = \int_0^1 \sqrt{\frac{1-\frac{1-t^2}{1+t^2}}{1+\frac{2t}{1+t^2}}} \cdot \frac{2}{1+t^2}  dt$ $= \int_0^1 \sqrt{\frac{2t^2}{(1+t)^2}} \cdot \frac{2}{1+t^2}  dt = 2\sqrt{2} \int_0^1 \frac{t}{(1+t)(1+t^2)}  dt$ $\frac{t}{(1+t)(1+t^2)} = \frac{A}{1+t} + \frac{Bt+C}{1+t^2}$ Hence $t \equiv A(1+t^2) + (Bt+C)(1+t)$ From which $A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$ Int is $2\sqrt{2} \left[ -\frac{1}{2}\ln(1+t) + \frac{1}{4}\ln(1+t^2) + \frac{1}{2}\tan^{-1}t \right]_0^1$ $= \frac{1}{4}(\pi - 2\ln 2)\sqrt{2}$	B1 M1 B1 A1 B1 M1 B1 A1 B1√ B1√ M1 A1	4	For this relation, stated or used For complete substitution for <i>x</i> in integrand For justification of limits 0 and 1 for <i>t</i> For correct simplification to given answer For statement of correct form of pfs For any use of the identity involving <i>B</i> or <i>C</i> For correct value of <i>A</i> For correct value of <i>B</i> For correct value of <i>C</i> For both logarithm terms correct For the inverse tan term correct For use of appropriate limits For correct (exact) answer in any form